Connectivity and rate with physical layer security over boundaries

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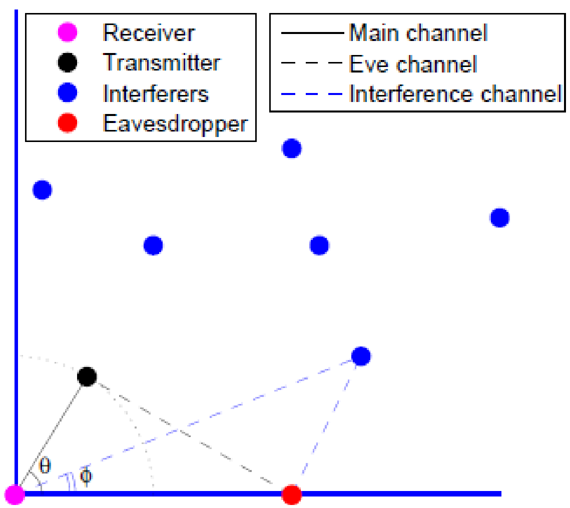
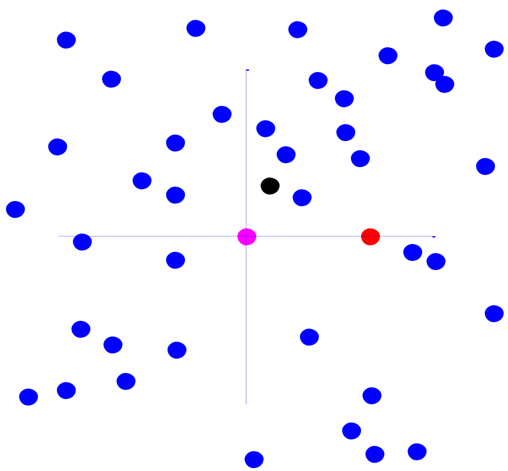
Motivation

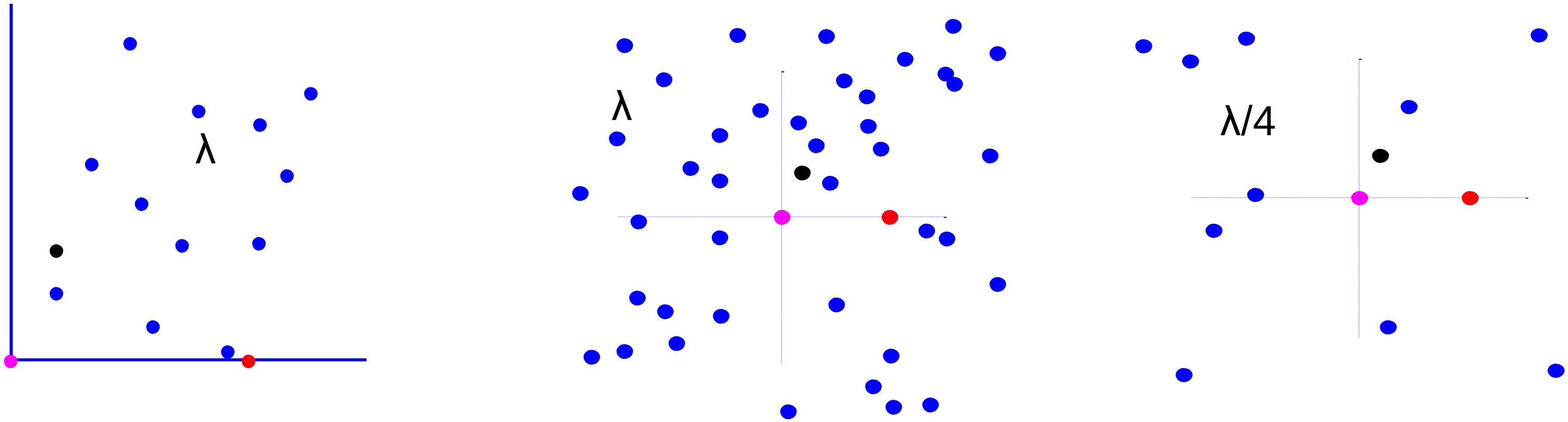
- Two deployment scenarios; Corner vs. Bulk
- Single eavesdropper - fixed and known location
- When it becomes beneficial to hide the receiver at the corner?





Motivation

- At the corner the mean and variance are scaled by ¼
- Impact of interferer's intensity in the bulk

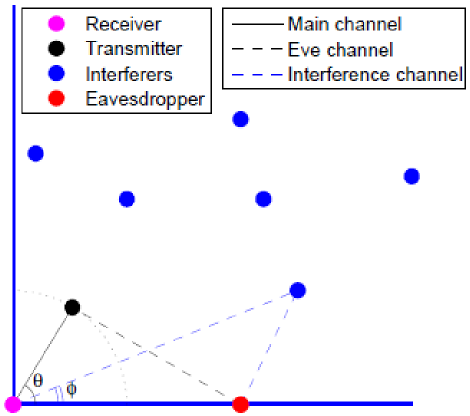


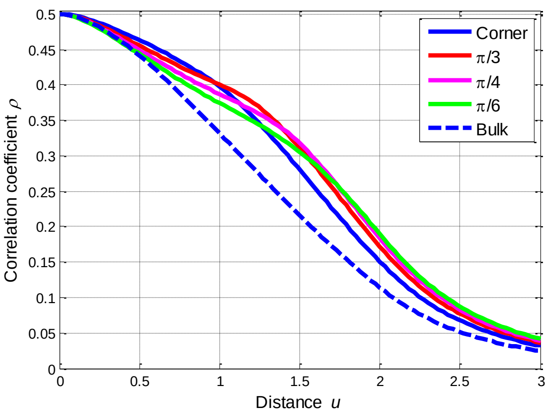
- How the performance at the corner with intensity of interferers λ differs from the performance in the bulk with scaled intensity (λ/4) of interferers?
  - Interference at the receiver and the eavesdropper
  - Spatial correlation of interference

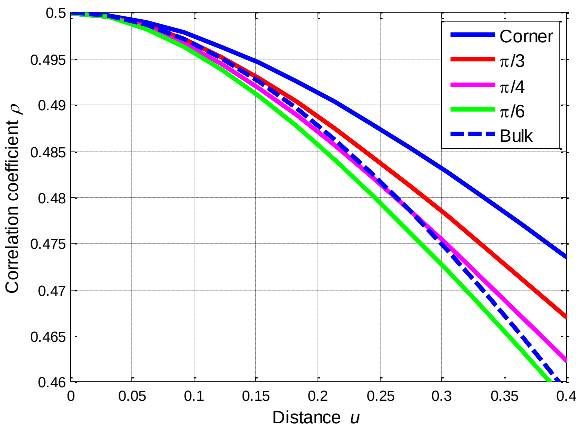
Spatial correlation of interference

- Spatial correlation of interference is independent of the user density

$$\rho_{\mathbf{x}}(u) = \frac{\lambda\xi \int_0^{\infty} \int_0^{\phi_{\mathbf{x}}} g(r)\, g\left(\|re^{j\phi} - u\|\right) r \mathrm{d}\phi\, \mathrm{d}r}{\sqrt{\mathbb{V}\mathrm{ar}\left\{\mathcal{I}_{\mathbf{x}}(u)\right\}}\sqrt{\mathbb{V}\mathrm{ar}\left\{\mathcal{I}_{\mathbf{x}}\right\}}}$$







- Spatial correlation of interference is higher when the receiver is located at the corner – trade off

Physical layer security

- Wyner encoding scheme
  - R_t is the rate of the transmitted codewords
  - R_s is the rate of confidential messages
  - R_e=R_t-R_s is the rate cost for securing the message against eavesdropping
- Unknown CSI
  - The rates R_t and R_s are kept fixed
  - SIR associated with the rates $\mu = 2^{R_t} - 1 \quad \sigma = 2^{R_e} - 1$
  - Probability of secure connectivity $\mathbb{P}_{\mathrm{x}}^{\mathrm{sc}}(u) = \mathbb{P}\left(\gamma_{\mathrm{x,r}} > \mu, \gamma_{\mathrm{x,e}}(u) < \sigma\right)$
- Known CSI
  - AMC based on the instantaneous SIR
  - Average secrecy rate describes the performance

$$\overline{C}_{\mathrm{x}}^{\mathrm{sc}}(u) = \int_0^\infty \int_0^{\gamma_{\mathrm{x,r}}} \log_2\left(\frac{1+\gamma_{\mathrm{x,r}}}{1+\gamma_{\mathrm{x,e}}}\right) f_{\mathrm{r,e}}(\gamma_{\mathrm{x,r}}, \gamma_{\mathrm{x,e}})\, \mathrm{d}\gamma_{\mathrm{x,e}} \mathrm{d}\gamma_{\mathrm{x,r}}$$

Probability of secure connectivity – unknown CSI

- Probability of secure connectivity with correlated interference

$$\mathbb{P}_{\mathrm{x}}^{\mathrm{sc}}(u) = \mathbb{P}\left(\gamma_{\mathrm{x,r}} > \mu, \gamma_{\mathrm{x,e}}(u) < \sigma\right)$$

$$= \mathbb{E}\left\{e^{-s\mathcal{I}_{\mathrm{x,r}}}\left(1 - e^{-s_e\mathcal{I}_{\mathrm{x,e}}(u)}\right)\right\} = \mathbb{P}_{\mathrm{x,r}}^{\mathrm{c}} - \mathcal{J}_{\mathrm{x}}(u)$$

$$\mathcal{J}_{\mathrm{x}}(u) = \mathbb{E}\left\{e^{-s\mathcal{I}_{\mathrm{x,r}} - s_e\mathcal{I}_{\mathrm{x,e}}(u)}\right\}$$

$$= \int_0^{\frac{\pi}{2}} \exp\left(-\lambda \int_{S_{\mathrm{x}}} \left(1 - \frac{1}{1 + sg(r)} \frac{1}{1 + s_e g(d)}\right) \mathrm{d}S\right) f_\Theta \mathrm{d}\theta$$

$$= \int_Z \exp\left(-\lambda \int_{S_{\mathrm{x}}} \left(1 - \frac{1}{1 + \mu g(r)} \frac{1}{1 + \sigma z^{-1} g(d)}\right) \mathrm{d}S\right) f_Z \mathrm{d}z$$

Bulk vs. Corner at low transmission rates R_t

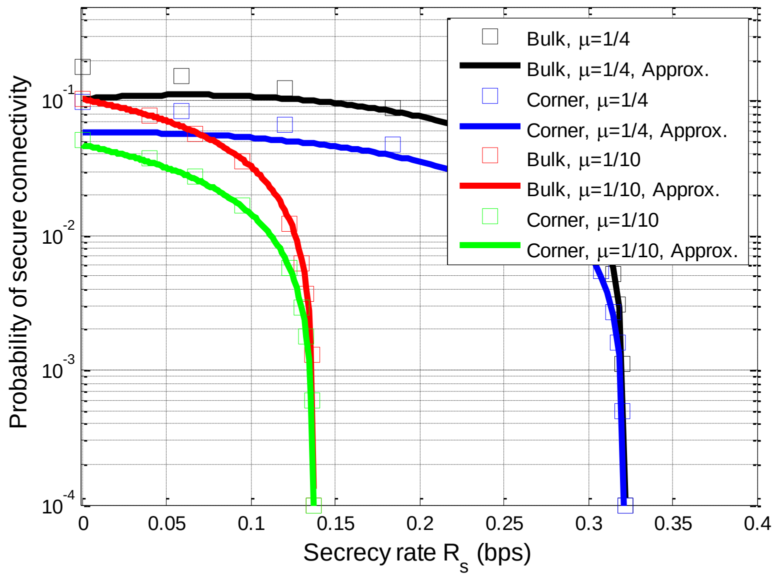
- Expanding the probability of secure connectivity for low *μ*, *σ*
- Connection probability

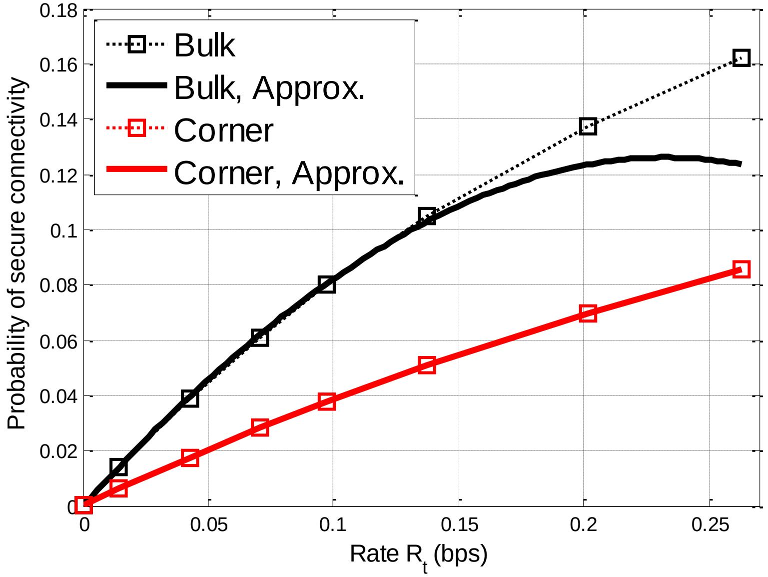
$$\begin{aligned}\mathbb{P}^{c}_{\mathrm{x,r}} &= \exp\left(-\lambda \int_0^\infty \int_0^{\phi_{\mathrm{x}}} \frac{sg(r)}{1+sg(r)} r \,\mathrm{d}\phi\, \mathrm{d}r\right)\\ &\approx \exp\left(-\lambda\left(\int_{S_{\mathrm{x}}} \left(\mu g(r) - \mu^2 g^2(r)\right) \mathrm{d}S\right)\right)\\ &= \exp\left(-\mu \mathbb{E}\{\mathcal{I}_{\mathrm{x,r}}\} + \frac{\mu^2}{2}\mathbb{V}\mathrm{ar}(\mathcal{I}_{\mathrm{x,r}})\right)\\ &\approx 1 - \mu\mathbb{E}\{\mathcal{I}_{\mathrm{x,r}}\} + \frac{\mu^2}{2}\left(\mathbb{V}\mathrm{ar}(\mathcal{I}_{\mathrm{x,r}}) + \mathbb{E}\{\mathcal{I}_{\mathrm{x,r}}\}^2\right)\end{aligned}$$

- Similarly we can expand $\mathcal{J}_{\mathrm{x}}(u)$
- Probability of secure connectivity at low-rate transmissions is proportional to the mean interference at the eavesdropper

Bulk vs. Corner at low transmission rates R_t

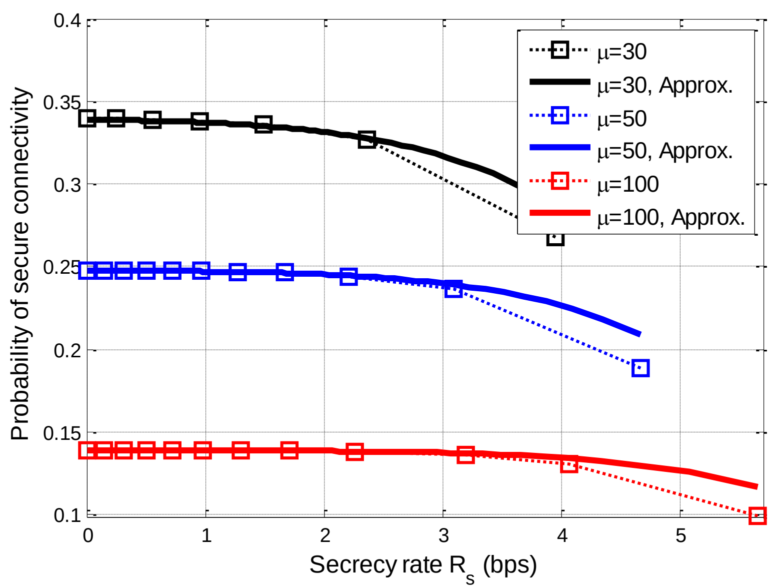
1. Mean interference along the boundary is always less than mean interference in the bulk → Bulk is preferable for low rate transmissions
2. The impact of interference correlation vanishes at low rates

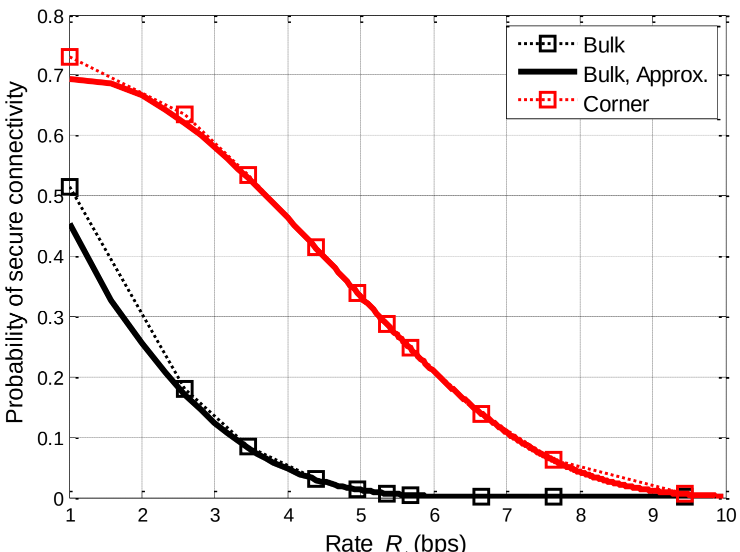




Bulk vs. Corner at high transmission rates R_t

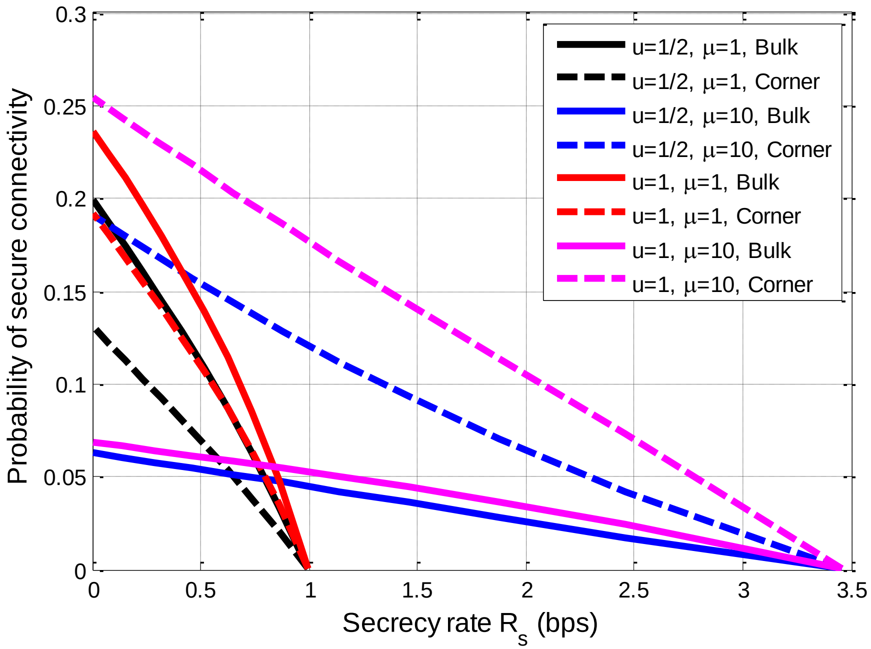
3. For large distance separation *u* between the receiver and the eavesdropper, placing the receiver at the corner is preferable at high transmission rates R_t
   - Assume independent interference and expand for high μ, σ

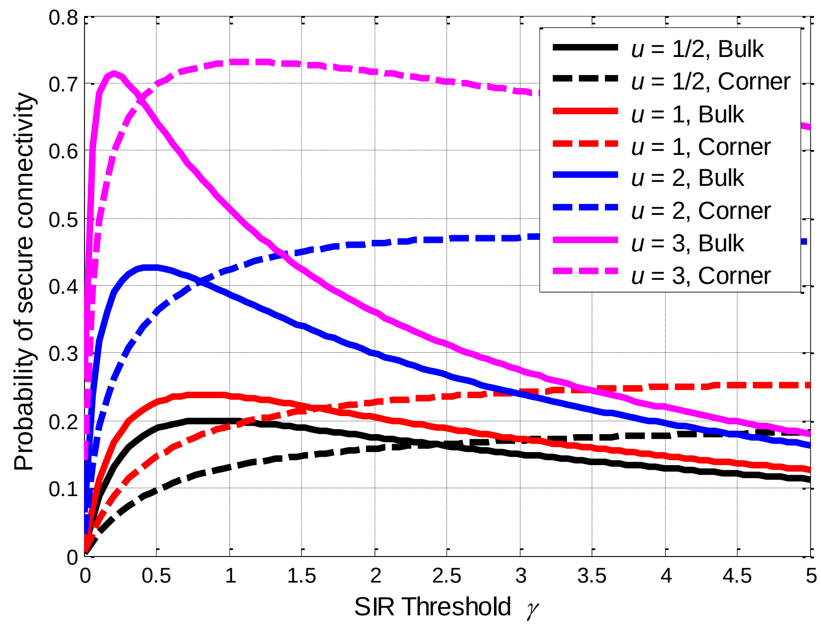




Bulk vs. Corner at high transmission rates R_t

4. For small distance separation *u* between the receiver and the eavesdropper, placing the receiver at the corner is still preferable



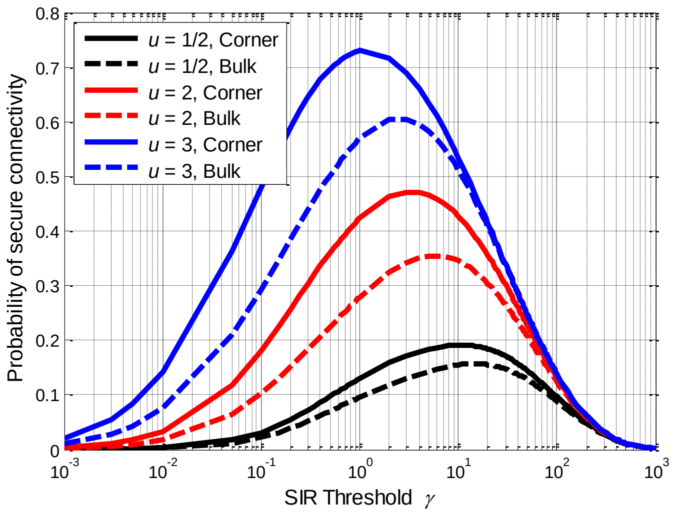


Average secrecy rate – known CSI

- Physical layer security reduces the average rate by a quantity that depends on the joint connection probability of receiver and eavesdropper for *μ=σ=γ*

$$\overline{C}_{\mathrm{x}}^{\mathrm{sc}}(u) = \overline{C}_{\mathrm{x}} - \frac{1}{\log(2)} \int_0^\infty \frac{\mathcal{J}_{\mathrm{x}}(u,\gamma)}{1+\gamma} \mathrm{d}\gamma$$

5. The average secrecy rate at the corner is higher than in the bulk even if the density of interferers over there is 4 times higher than in the bulk



Future work

- Point Process for the eavesdroppers
- Interference correlation in more complex geometries
- Performance of secrecy enhancement techniques