A comparison of nonlinear noise reduction and independent component analysis using a realistic dynamical model of the electrocardiogram

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ABSTRACT

Accurate performance metrics for removing noise from the electrocardiogram (ECG) are difficult to define due to the inherently complicated nature of the noise and the absence of knowledge about the underlying dynamical processes. By using a previously published model for generating realistic artificial ECG signals and adding both stochastic and deterministic noise, a method for assessing the performance of noise reduction techniques is presented. Independent component analysis (ICA) and nonlinear noise reduction (NNR) are employed to remove noise from an ECG with known characteristics. Performance as a function of the signal to noise ratio is measured by both a noise reduction factor and the correlation between the cleaned signal and the original noise-free signal.

Keywords: Electrocardiogram, nonlinear noise reduction, independent component analysis, noise, measurement error.

1. INTRODUCTION

The field of biomedical signal processing has given rise to a number of techniques for assisting physicians with their everyday tasks of diagnosing and monitoring medical disorders. The electrocardiogram (ECG) provides a quantitative description of the heart's electrical activity. It is a time-varying signal that reflects the ionic current flow, which causes the cardiac muscles to contract and relax during each heart beat. The surface ECG is obtained by measuring the potential difference between two electrodes placed on the skin. The ECG is routinely used in hospitals throughout the world as a tool for identifying cardiac disorders.

A large variety of signal processing techniques have been employed for transforming the raw ECG signal into a tool for diagnosing and monitoring medical disorders. A typical ECG is invariably corrupted by (i) electrical interference from surrounding equipment (*e.g.* effect of the electrical mains supply), (ii) measurement noise, (iii) analogue to digital conversion and (iv) movement artefacts. Many techniques exist for filtering[1] and removing noise from the raw ECG signal, such as Principal Component Analysis (PCA),[2] Independent Component Analysis (ICA)[3] and nonlinear noise reduction.[4] The ECG signal is fundamental for numerous medical studies, including the investigation of heart rate variability, respiration and QT dispersion. The utility of these medical indicators relies on signal processing techniques for detecting R-peaks,[5,6] deriving heart rate and respiratory rate,[7] and measuring QT-intervals.[8]

In order to employ the ECG signal for facilitating medical diagnosis, biomedical processing techniques are generally used to *clean* the signal, thereby attempting to remove some or all of the above sources of noise. Available spectral techniques include notch filters for removing the effect of the electrical mains supply, and both low and high band pass filters for removing noise that dominates the high and low frequencies respectively. In many cases, the availability of more than one lead for recording the ECG offers the opportunity to construct a clean signal by taking averages over the different leads. A technique known as principal component analysis (PCA) is one approach, which uses projections onto an orthogonal basis set to separate the underlying signal

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from the noise. It assumes that the directions capturing most of the variance also contain large amounts of the signal; this is not necessarily true since important parts of the signal may have a variance similar to that of the noise. Nonlinear noise reduction[4] has also been employed to filter ECG signals; this technique is capable of separating the underlying signal from noise even in cases where both the noise and parts of the signal occupy similar regions of the frequency domain. Independent component analysis (ICA) aims to separate a collection of signals into a group of independent sources; the idea being that one will capture the underlying signal and that the others will contain various sources of noise. It is also possible to apply ICA to a single univariate ECG signal by first employing a delay-coordinate transformation[9] to reconstruct a state space for representing the underlying dynamics.

It is difficult to evaluate the accuracy of noise reduction and filtering techniques applied to the ECG since it is impossible to have access to a truly clean ECG signal. The fact that the true underlying dynamics of a real ECG can never be known implies that one cannot distinguish between the clean ECG signal and the many sources of noise that can occur during recording in a clinical environment. While the availability of biomedical databases[10] provides a useful benchmark for comparing techniques and approaches, they still fail to provide a means of testing a technique's ability to remove noise from the ECG signal. The recent availability of a dynamical model for generating ECG signals with known temporal and spectral characteristics and pre-specified average morphology[11] allows researchers to compare and evaluate a variety of signal processing techniques under different conditions as specified by their particular needs. The freely available open-source software (Matlab and C code) and a Java applet[12] for downloading data directly from the Internet can be used to specify a *gold standard*, whereby an ECG with well-understood dynamics can be employed for testing new signal processing techniques.

2. METHODS

2.1. Artificial ECG signal

A dynamical model for generating artificial ECG signals with pre-specified temporal and spectral properties forms the basis of this investigation.[11] This model may be implemented using freely available open-source software[12] and may also be operated directly over the Internet using a Java applet. The underlying dynamical system uses a three-dimensional state space to reproduce the relevant movements reflected in the ECG. A limit cycle in the (x, y)-plane provides the quasi-periodic motion during each cycle of the heart. A series of Gaussian functions forces the z-component to trace out an average ECG morphology consisting of the P,Q,R,S and T peaks. Both the height and width of each of these peaks may be specified. The beat-to-beat heart rate and associated RR intervals are controlled by specifying an internal time series with chosen spectral characteristics; position and width of both the low frequency (LF) and high frequency (HF) peaks in the power spectrum and the LF/HF ratio may be selected. This internal time series is used to drive the angular frequency of motion around the limit cycle, thereby transferring the specified spectral characteristics to the RR intervals of the ECG. This model has also been extended to produce realistic coupled nonlinear artificial ECG, blood pressure and respiratory signals.[13]

The ECG signal used throughout this investigation had a mean heart rate of 60 beats per minute, a heart rate standard deviation of 1 beat per minute and a sampling frequency of 256 Hz. The positions of the LF and HF components of the spectrum were 0.1 Hz and 0.25 Hz respectively and both had a standard deviation of 0.01 Hz. The LF/HF ratio was 0.5.

2.2. Noise

Noise is an all-encompassing term used to describe uncertainty in the data or specifically the part of the data that does not directly reflect the underlying system of interest. Sources of noise commonly encountered in the ECG include (i) electrical interference from surrounding equipment (*e.g.* effect of the electrical mains supply), (ii) measurement noise, (iii) analogue to digital conversion and (iv) movement artefacts.

There is an important distinction to be made between *observational uncertainty* and *dynamical uncertainty*. Observational uncertainty refers to measurement errors which are independent of the dynamics. Sources include finite precision measurements, truncation errors, and missing data (both temporal and spatial). In contrast, dynamical uncertainty refers to external fluctuations interacting with and changing internal variables in the underlying system. While observational uncertainty obscures the state vectors, dynamical uncertainty changes the actual dynamics.

This paper considers the effects of noise due to observational uncertainty. The simplest description of observational uncertainty is additive measurement error where the recorded signal $y(t)$ is given by

$$y(t) = x(t) + \epsilon(t), \tag{1}$$

where $x(t)$ is the true state vector and $\epsilon(t)$ represents the unobserved measurement error. This measurement error term is usually described by a random variable, for example an identically and normally distributed (IND) process, $\epsilon \sim N(0, \sigma^2_{\text{noise}})$, where σ^2_{noise} is the variance of the noise. If the variance of the signal is σ^2_{signal}, then the *signal to noise ratio* is defined as

$$\gamma = \frac{\sigma_{\text{signal}}}{\sigma_{\text{noise}}}. \tag{2}$$

2.3. Evaluation

After applying a technique for *cleaning* the noisy data, a measure of the success of this procedure is required. Let the cleaned signal be $z(t)$. Following Schreiber and Kaplan,[4] a *noise reduction factor*,

$$\chi = \sqrt{\frac{\langle y(t) - x(t)\rangle^2}{\langle z(t) - x(t)\rangle^2}}, \tag{3}$$

where $\langle\cdot\rangle$ denotes the average over time t, is employed to provide a measure of the factor by which the RMS error is reduced. Unlike the investigation of Schreiber and Kaplan,[4] the ECG model may be used to obtain a truly noise-free signal $x(t)$ so that the value of χ may be viewed as the actual noise reduction factor and not merely a lower bound. The higher the value of χ, the better the noise reduction procedure, whereas $\chi = 1$ indicates no improvement since similar accuracy could have been achieved by using the noisy signal, $y(t)$, instead of $z(t)$.

An alternative measure of noise reduction performance is given by a measure of the linear correlation between the cleaned signal, $z(t)$, and the original noise-free signal, $x(t)$. The correlation coefficient ρ between two signals $x(t)$ and $z(t)$ is given by[14]

$$\rho = \frac{\langle [x(t) - \mu_y][z(t) - \mu_z]\rangle}{\sigma_x \sigma_z}, \tag{4}$$

where μ_x and σ_x are the mean and standard deviation of x and μ_z and σ_z are the mean and standard deviation of z. Values of $\rho \sim 1$ reflect strong linear correlations, $\rho \sim -1$ implies strong linear anti-correlations, and $\rho \sim 0$ indicates that no linear correlations exist. Therefore a value of $\rho = 1$ would suggest that the noise reduction technique has removed all the noise from the observed signal.

2.4. State space reconstruction

Reconstructing a state space using an observed time series is usually the first step towards building a model for describing nonlinear dynamics. Suppose that the underlying system dynamics of the ECG evolve on an attractor $\mathbf{A}$ according to $\mathbf{G} : \mathbf{A} \to \mathbf{A}$. Let τ_s be the sampling time of the recorded signal. Given an observed time series, $y_i = y(i\tau_s)$, recorded by a measurement function, $\mathbf{h} : \mathbf{A} \to \mathbb{R}$ of the system state space variables, it is possible to construct a replica state space using a delay vector reconstruction[9,15,16] of the observed time series y_i defined by

$$\mathbf{v}_i = \left[y_i, y_{i+d} \ldots, y_{i+(m-1)d}\right] \in \mathbb{R}^m \tag{5}$$

where m is the reconstruction dimension and $\tau_d = d\tau_s$ is the time delay. In order to reconstruct the dynamics of the system state space, $\mathbf{G}$, using a data-driven model, $\mathbf{F} : \mathbf{\Phi}(\mathbf{A}) \to \mathbf{\Phi}(\mathbf{A})$, it is necessary to ensure that the mapping $\mathbf{\Phi} : \mathbf{A} \to \mathbf{\Phi}(\mathbf{A})$ provides a faithful representation of the system's attractor; in mathematical terms, $\mathbf{\Phi}$ must provide an embedding.[17] Values for τ_d and m may be determined by optimising the accuracy of the noise reduction techniques.

2.5. Nonlinear noise reduction

The ECG signal cannot be classified as either periodic or deterministically chaotic. While the ECG signal is not predictable in the long term, it displays limited predictability over times less than one heart beat. Schreiber and Kaplan[4] successfully applied a technique, originally constructed for removing noise from chaotic signals,[18] to ECG signals. This short-term predictability may be used to reduce measurement errors by a local geometric method. The basic idea behind this so-called nonlinear noise reduction is to use the manifold of the underlying dynamical system to project out the noise. This may be achieved by using a local linear model to predict a particular point in the state space while using its neighbours (both backwards and forwards in time) to construct a local linear map. This process may be iterated a number of times to clean the entire time series. For the ECG, best results are obtained using only one iteration.[4] This type of filtering is nonlinear in the sense that the effective filter given by the local linear map varies throughout state space depending on the local dynamics. In particular it has the ability to remove noise from the recorded data, even in cases when the underlying signal and the noise overlap in the frequency domain.

The availability of a noise-free artificial ECG implies that the correct answer is known and that a thorough search of the parameter space is possible. The nonlinear noise reduction technique, known as *nrlazy* (see[19,20]), requires the choice of various parameters such as the reconstruction dimension, m, the time delay, d and the neighbourhood radius, r.

2.6. Independent component analysis

Independent Component Analysis (ICA) is a statistical technique for decomposing a dataset into independent sub-parts.[3,21] Using the delay reconstruction described in section 2.4, the observed univariate ECG signal, $y_i = y(i\tau_s)$, is transformed into an $m \times n$ embedding matrix,

$$\mathbf{Y} = \begin{bmatrix} y_1 & y_{1+d} & \cdots & y_{1+(n-1)d} \\ y_{1+d} & y_{1+2d} & \cdots & y_{1+nd} \\ \vdots & \vdots & & \vdots \\ y_{1+(m-1)d} & y_{1+md} & \cdots & y_{1+(m+n-2)d} \end{bmatrix}, \tag{6}$$

where each column of $\mathbf{Y}$ contains one reconstructed state vector as defined by equation (5). Note that the observed ECG signal, y_i, is assumed to be mean zero with unit standard deviation, achieved by removing the mean, μ_y, of y and dividing by its standard deviation, σ_y. After application of the ICA algorithm, the resulting cleaned signal is rescaled by multiplying by σ_y and adding μ_y so that it is compatible with y_i.

In mathematical terms, the problem may be expressed as

$$\mathbf{Y} = \mathbf{BX}, \tag{7}$$

where $\mathbf{X}$ is an $m \times n$ matrix containing the independent source signals, $\mathbf{B}$ is the $m \times m$ mixing matrix, and $\mathbf{Y}$ is an $m \times n$ matrix containing the observed (mixed) signals. ICA algorithms attempt to find a separating or de-mixing matrix $\mathbf{W}$ such that

$$\mathbf{X} = \mathbf{WY}. \tag{8}$$

In practice, iterative methods are used to maximise or minimise a given cost function such as mutual information, entropy or the kurtosis (fourth order cumulant), which is given by

$$kurt(\mathbf{Y}) = E\{\mathbf{Y}^4\} - 3(E\{\mathbf{Y}^2\})^2 \tag{9}$$

where $E\{\mathbf{Y}\}$ is the expectation of $\mathbf{Y}$. The following analysis uses Cardoso's Multidimensional ICA algorithm *jadeR*,[3] which is based upon the joint diagonalisation of cumulant matrices, because it combines the benefits of both PCA and ICA to provide a stable deterministic solution. (ICA suffers from a scaling and column ordering problem due to the indeterminacy of solution to scalar multipliers to and column permutations of the mixing matrix).

Most ICA methods assume there are at least as many independent measurement sensors as sources one wishes to separate. Following the method of James *et al.*[21] to perform ICA blind source separation of the embedding

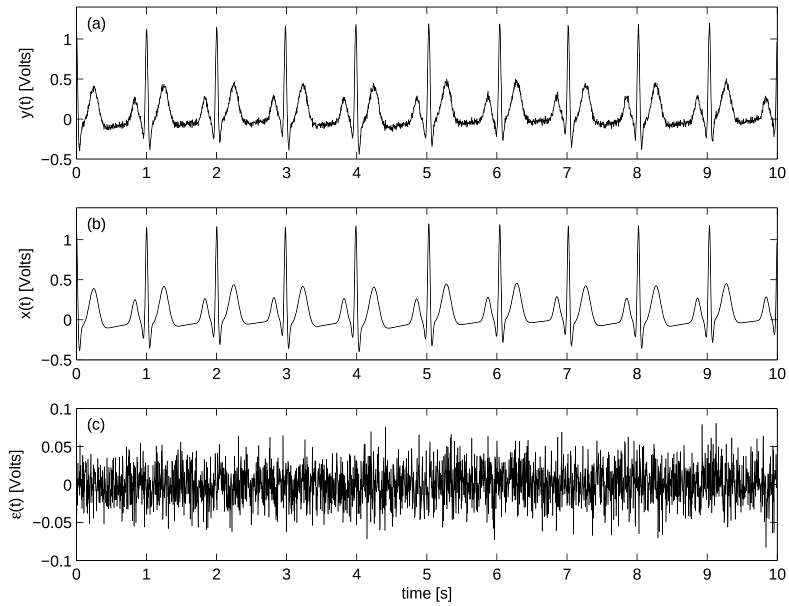

Figure 1. Effect of additive IND measurement errors: (a) artificial ECG signal with additive stochastic measurement noise $y(t)$, (b) noise-free artificial ECG signal $x(t)$ and (c) measurement errors $\epsilon(t)$. The signal to noise ratio is $\gamma = 10$.

matrix $\mathbf{Y}$, the assumption of one signal and one noise source is employed with Cardoso's *jadeR* algorithm. An estimate $\hat{\mathbf{X}}$ of the sources $\mathbf{X}$ are obtained from

$$\hat{\mathbf{X}} = \mathbf{WY} \tag{10}$$

where $\hat{\mathbf{X}}$ is the ICA estimate of $\mathbf{X}$. and $[\cdots]_{maxC}$ denotes the (normalised) vector that maximally correlates with the normalised version of $\mathbf{X}$. Note that due to the scaling and inversion indeterminacy problem of ICA, both $\pm$ each row of $\mathbf{X}$ must be considered. The scaling problem is addressed by multiplying by σ_y and adding μ_y. The row with the highest correlation with the original noise-free signal is chosen as the best estimate, $z(t)$, of noise-free signal $x(t)$.

3. RESULTS

The application of both the nonlinear noise reduction (NNR) and independent component analysis (ICA) techniques is now considered for two separate cases: (i) stochastic noise and (ii) deterministic noise.

3.1. Stochastic noise

In this section, the noise is assumed to be additive measurement errors represented by a normal distribution with zero mean. Signal to noise ratios of $\gamma = 10, 5, 2.5$ were considered and in each case NNR gave optimal results for a delay of $d = 1$. The effect of IND additive measurement errors with $\gamma = 10$ are shown in Fig. 1.

For a signal to noise ratio of $\gamma = 10$, the NNR technique gives an optimal noise reduction factor of $\chi = 2.09$ for $m = 16$ and $r = 0.08$ (Fig. 2). A closer examination of the dependence of χ on m is obtained by taking a cross-section of the surface shown in Fig. 2 at $r = 0.08$. As shown in Fig. 3a there is a maximum noise reduction factor of $\chi = 2.2171$ at $m = 20$. The various time series and the error involved in the noise reduction process are illustrated in Fig. 4. This shows that while the cleaned signal, $z(t)$ (Fig.4c) closely resembles the original noise-free signal, $x(t)$, (Fig. 4b) there still remains considerable structure in the error, $z(t) - x(t)$ (Fig. 4d). This structure is particularly evident and appears larger around the QRS complex. As pointed out by Schreiber and Kaplan,[4] the NNR technique attempts to minimise the resulting RMS error and does not directly aim to recover other key characteristics of the ECG that may be of more clinical relevance to the physician. Despite

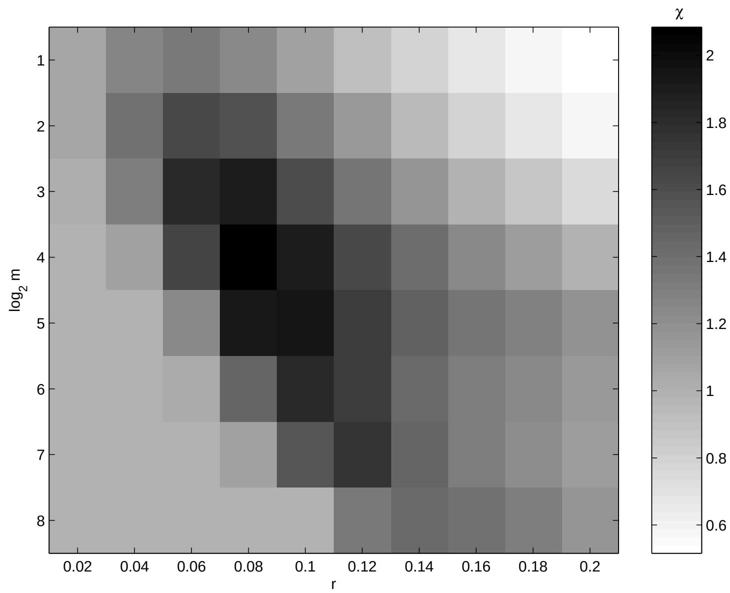

Figure 2. Variation of noise reduction factor, χ with reconstruction dimension, m, and neighbourhood size, r, for NNR with data having a signal to noise ratio of $\gamma = 10$.

Table 1. Noise reduction performance in terms of noise reduction factor, χ and correlation, ρ for both nonlinear noise reduction (NNR) and independent component analysis (ICA) for three signal to noise ratios, $\gamma = 10, 5, 2.5$.

Method	Measure	$\gamma = 10$	$\gamma = 5$	$\gamma = 2.5$
NNR	χ	2.2171	2.6605	3.3996
ICA	χ	26.7265	18.9325	10.8842
NNR	ρ	0.9990	0.9972	0.9939
ICA	ρ	0.9980	0.9942	0.9845

this, the NNR technique does recover the peaks and troughs that define the morphology of the ECG. Both the P-waves and T-waves are clearly visible in Fig. 4c and there positions and magnitudes remain faithful to that of the original noise free ECG in Fig. 4b.

The NNR technique gives optimal results for neighbourhood sizes of different sizes depending on the signal to noise ratio: (i) $r = 0.08$ for $\gamma = 10$, (ii) $r = 0.175$ for $\gamma = 5$ and (iii) $r = 0.4$ for $\gamma = 2.5$. Figure 3 shows both the noise reduction factor, χ, and the correlation, ρ, as a function of the reconstruction dimension, m for signals having $\gamma = 10, 5, 2.5$. For $\gamma = 10$, maxima occur at $\chi = 2.2171$ and $\rho = 0.9990$ are at $m = 20$. For $\gamma = 5$, maxima occur at $\chi = 2.6605$ and $\rho = 0.9972$ are at $m = 20$. For $\gamma = 2.5$, the noise reduction factor has a maximum, $\chi = 3.3996$ at $m = 100$ whereas the correlation, $\rho = 0.9939$ has a maximum at $m = 68$.

ICA gave best results for all signal to noise ratios for a delay of $d = 1$. As may be seen from Fig, 5, optimising over the noise reduction factor, χ, or the correlation, ρ, gave maxima at different values of m. For $\gamma = 10$, the maxima are $\chi = 26.7265$ at $m = 7$ and $\rho = 0.9980$ at $m = 9$. For $\gamma = 5$, the maxima are $\chi = 18.9325$ at $m = 7$ and $\rho = 0.9942$ at $m = 9$. For $\gamma = 2.5$, the maxima are $\chi = 10.8842$ at $m = 8$ and $\rho = 0.9845$ at $m = 11$. A demonstration of the effect of optimising the ICA algorithm over either χ or ρ is illustrated in Fig. 6. While both the χ-optimised cleaned signal (Fig. 6b) and the ρ-optimised cleaned signal (Fig. 6d) are similar to the original noise-free signal (Fig. 6a), an inspection of their respective errors, (Fig. 6c) and (Fig. 6e), emphasises their differences. The χ-optimised outperforms the ρ-optimised in recovering the R-peaks.

The results of both the NNR and ICA techniques are given in Table 1, showing that NNR performs better

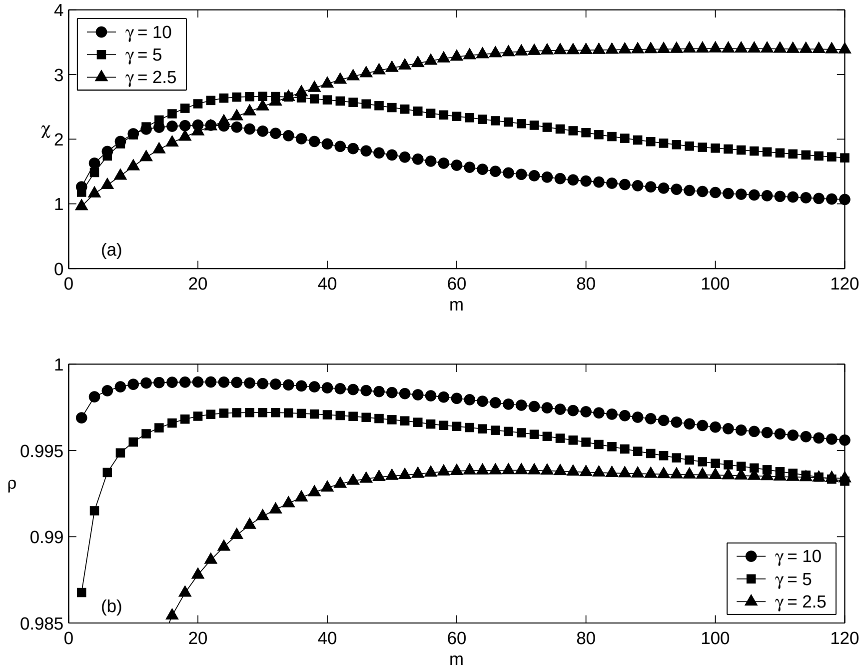

Figure 3. Variation in (a) noise reduction factor, χ, and (b) correlation, ρ, with reconstruction dimension, m, for NNR applied to data with signal to noise ratios of $\gamma = 10$ (•), $\gamma = 5$ (■) and $\gamma = 2.5$ (▲), having neighbourhood sizes of $r = 0.08, 0.175, 0.4$ respectively.

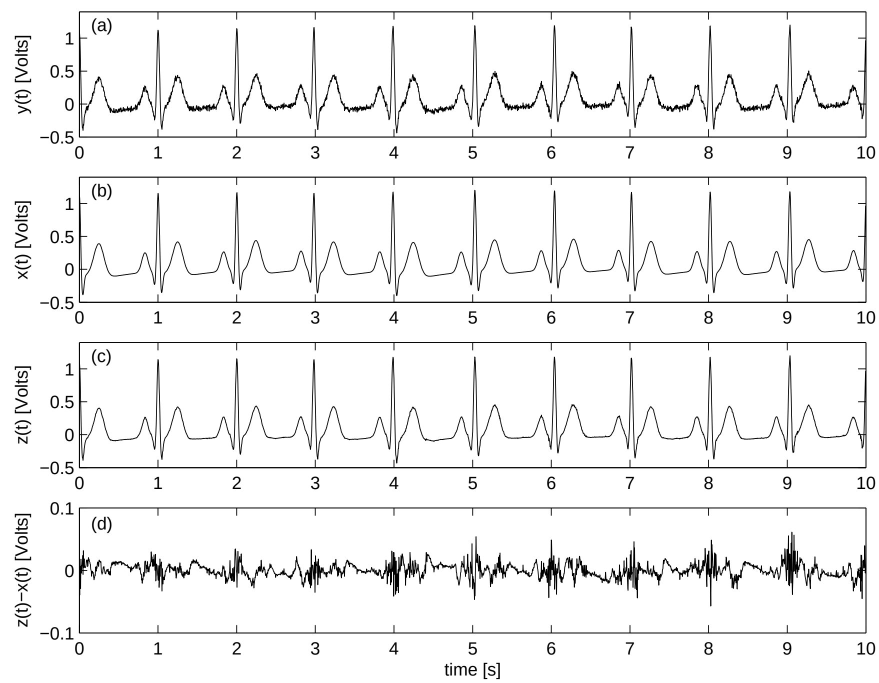

Figure 4. Demonstration of the nonlinear noise reduction: (a) original noisy ECG signal, $y(t)$, (b) underlying noise-free ECG, $x(t)$ and noise-reduced ECG signal, $z(t)$ and (c) remaining error, $z(t) - x(t)$. The signal to noise ratio was $\gamma = 10$ and the NNR used parameters $m = 20$, $d = 1$ and $r = 0.08$.

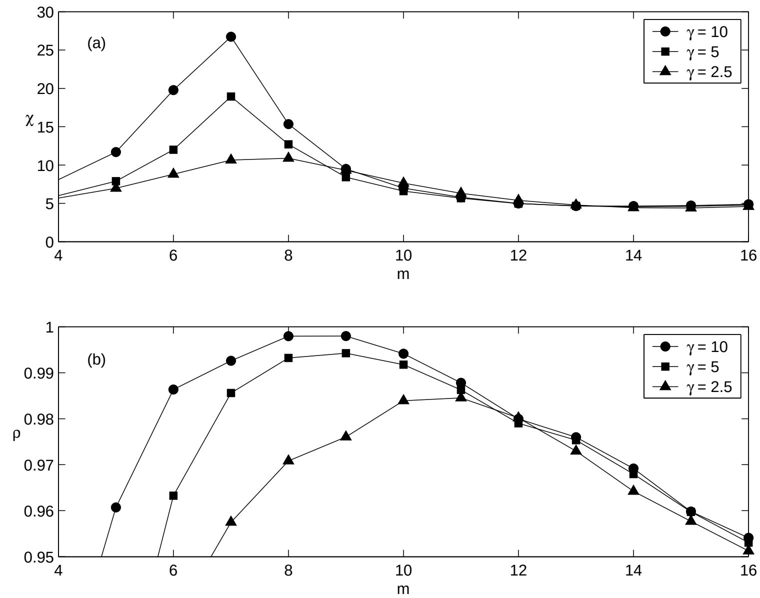

Figure 5. Variation in (a) noise reduction factor, χ, and (b) correlation, ρ, for ICA with reconstruction dimension, m and delay $d = 1$. The signal to noise ratios are $\gamma = 10$ (•), $\gamma = 5$ (■) and $\gamma = 2.5$ (▲).

in terms of providing a cleaned signal which is maximally correlated with the original noise-free signal, whereas ICA performs better in terms of yielding a cleaned signal which is closer to the original noise-free signal in a RMS sense. Whether an optimal χ or ρ is to be preferred depends on what the ECG is to be used for. If the morphology of the ECG is of importance and the various waves (P, QRS, T) are to be detected, then perhaps a large value of ρ is of greater value. In contrast, if the ECG is to be used to derive RR intervals, then the location in time of the R-peaks are required. In this case, the noise reduction factor may be preferable since it penalises heavily for large squared deviations and therefore will favour more accurate recovery of extrema such as the R-peak.

3.2. Deterministic noise

This section considers the removal of real artefact which may be represented by deterministic noise. For example, a transient repetitive finger motion, tapping on the sensor. This source of noise is simulated by a 4Hz sinusoid modulated by a hamming window. The maximum amplitude of this noise signal was fixed at 0.1 Volts. The effect of this deterministic noise on a clean ECG is shown in Fig. 7.

Again, both NNR and ICA performed best for a delay of $d = 1$. The NNR technique gave its best performance for a neighbourhood of $r = 0.08$. The ability of both NNR and ICA to remove this deterministic noise is quantified by the noise reduction factor, χ and the correlation, ρ as a function of the reconstruction dimension, m in Fig. 8. NNR provides a maximum of $\chi = 1.2264$ at $m = 22$ (Fig. 8a), whereas ICA is superior with $\chi = 16.4846$ at $m = 7$ (Fig. 8b). In contrast, NNR provides the best correlation results with $\rho = 0.9887$ at $m = 22$ (Fig. 8c) whereas ICA has $\rho = 0.9825$ at $m = 8$ (Fig. 8d). This analysis shows again that NNR outperforms ICA in terms of the achieving a maximum correlation, ρ, whereas ICA provides a better noise reduction factor, χ, than NNR.

4. CONCLUSION

An artificial electrocardiogram signal with controlled temporal and spectral characteristics has been employed to illustrate and compare the noise reduction performance of two techniques, nonlinear noise reduction (NNR) and independent components analysis (ICA). Both stochastic and deterministic sources of noise were simulated while investigating the accuracy of the two techniques for removing noise from the ECG signals. The stochastic noise

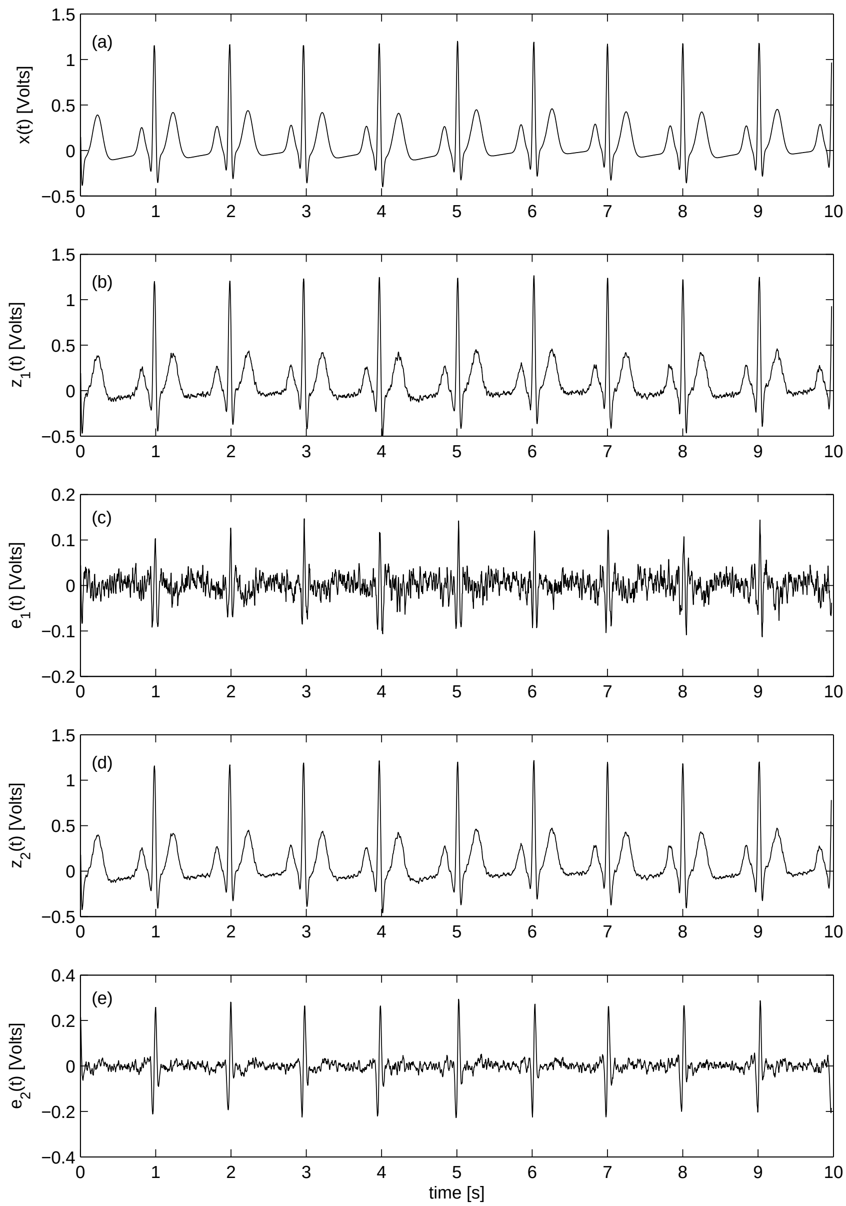

Figure 6. Demonstration of ICA noise reduction for $\gamma = 10$: (a) original noise-free ECG signal, $x(t)$, (b) χ-optimised noise-reduced signal, $z_1(t)$, with $m = 7$, (c) error, $e_1(t) = z_1(t) - x(t)$, (d) ρ-optimised noise-reduced signal, $z_2(t)$ with $m = 8$ and (e) error, $e_2(t) = z_2(t) - x(t)$.

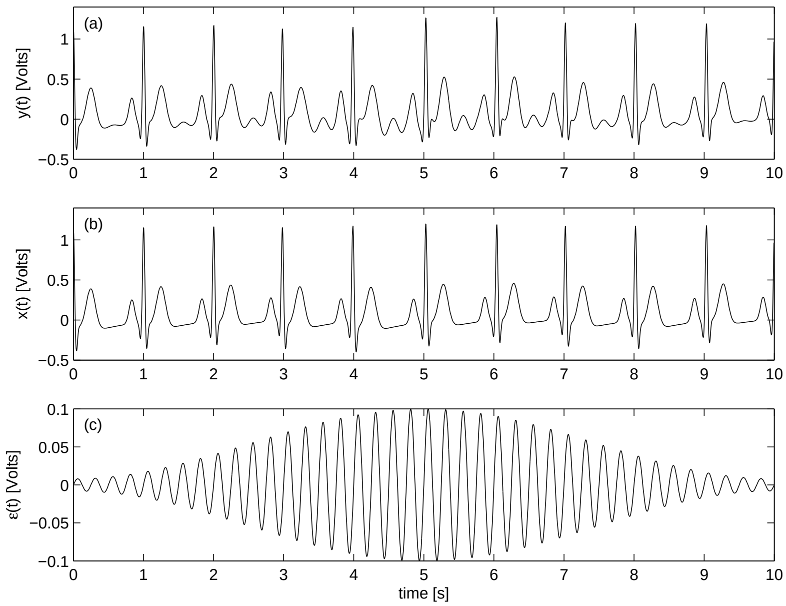

Figure 7. Effect of finger tapping noise: (a) artificial ECG signal with additive deterministic noise $y(t)$, (b) noise-free artificial ECG signal $x(t)$ and (c) finger tapping noise $\epsilon(t)$.

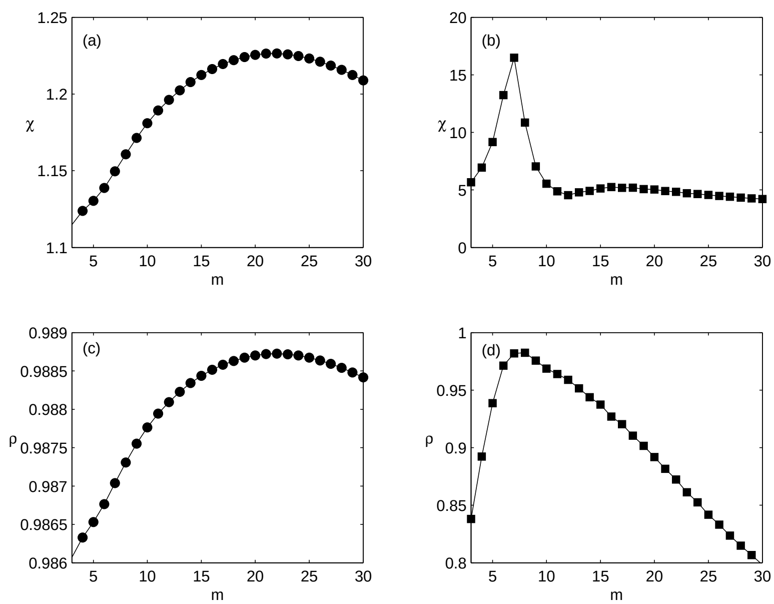

Figure 8. Removal of finger tapping noise. Variation in (a) NNR noise reduction factor, χ, (b) ICA noise reduction factor, χ, (c) NNR correlation, ρ and (d) ICA correlation, ρ with reconstruction dimension, m and delay $d = 1$.

simulates measurement errors and the latter represents artefacts in the ECG. The quality of the noise removal was evaluated by computing a noise reduction factor and the correlation between the cleaned signal and the original noise-free signal. In the case of both stochastic and deterministic noise, NNR was found to give better results as measured by correlation. In contrast, for both noise sources, ICA provided a cleaned signal with a larger noise reduction factor. These results suggest that NNR is superior at recovering the morphology of the ECG and is less likely to distort the shape of the P, QRS and T waves, whereas ICA is better at recovering specific points on the ECG such as the R-peak, which is necessary for obtaining RR intervals.

ACKNOWLEDGMENTS

PEM acknowledges support of a Research Fellowship from the Royal Academy of Engineering and the Engineering and Physical Sciences Research Council (EPSRC). GDC acknowledges support by the US National Institute of Health (NIH), grant number EC001659-01.

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