Effects of aberrations in spatiotemporal focusing of ultrashort laser pulses

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Spatiotemporal focusing, or simultaneous spatial and temporal focusing (SSTF), has already been adopted for various applications in microscopy, photoactivation for biological studies, and laser fabrication. We investigate the effects of aberrations on focus formation in SSTF, in particular, the effects of phase aberrations related to low-order Zernike modes and a refractive index mismatch between the immersion medium and sample. By considering a line focus, we are able to draw direct comparison between the performance of SSTF and conventional spatial focusing (SF). Wide-field SSTF is also investigated and is found to be much more robust to aberrations than either line SSTF or SF. These results show the sensitivity of certain focusing methods to specific aberrations, and can inform on the necessity and benefit of aberration correction. © 2014 Optical Society of America

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1. INTRODUCTION

Spatiotemporal focusing, or simultaneous spatial and temporal focusing (SSTF) has become a useful method in microscopy and laser fabrication, relying on the fact that a focused ultrashort pulse is a superposition of many spectral components in space time, which gives rise to a high instantaneous peak intensity. SSTF is realized by spatially separating the spectral components of a short pulsed laser beam across the illuminated pupil of a color corrected objective lens, thus ensuring that these components recombine into an ultrashort pulse only at the focus. SSTF has been applied to multiphoton microscopy for both harmonic generation and two-photon fluorescence imaging, in which high axial resolution and video-rate speed were demonstrated. There are extensive subsequent investigations using SSTF in imaging, such as adjusting group velocity dispersion for refocusing, imaging of biological samples, the creation of novel excitation patterns for biological photoactivation, as well as theoretical treatment. Concurrently SSTF has also been implemented in laser fabrication, where demonstrations include the generation of hollow microfluidic channels, control of the pulse front tilt (PFT) for directional laser writing, 3D lithographic microfabrication, patterned excitation, and longer pulse envelope fabrication.

In a perfect optical system, light focuses without distortion to a diffraction-limited focus, the form of which essentially determines the resolution of the microscope or laser fabrication system. However, inherent imperfections or misalignments in optical systems, refractive index differences between the objective immersion medium and specimen, and optical inhomogeneity in the specimen cause additional unwanted refraction of the light. This refraction introduces phase variations, or aberrations, which can severely affect the quality of imaging and laser fabrication, particularly when high NA objective lenses are employed. Thus, it is important to characterize how these aberrations affect the focusing properties, and subsequently develop methods for their compensation.

There have been numerous studies documenting the detrimental effects of aberrations in conventional spatial focusing (SF), which employs a spectrally uniform pupil illumination (the intensity distribution in the back focal plane of the objective lens) to create a single focal spot. It has been shown that phase aberrations may be compensated using adaptive optics elements (AOEs), such as liquid crystal spatial light modulators or deformable mirrors, either in microscopy or laser fabrication systems. However, until now, there has been no systematic analysis for the effects of aberrations in SSTF. In this paper, we study both the spatial and temporal effects of aberrations in SSTF and compare these with purely SF of pulsed lasers. Both line and wide-field foci are analyzed. The results are useful to illustrate the potential of aberration correction for SSTF microscopy, laser fabrication, and any similar applications.

2. PUPIL ILLUMINATION AND FOCUSING CONFIGURATION

Fourier optics is adopted in this paper to analyze the focusing properties of a lens under different illumination conditions. The intensity distribution in the focal plane can be obtained by solving the Rayleigh–Sommerfeld diffraction integral from the pupil illumination, which is represented by $E(ω, p_x, p_y)$, where $p_x$ and $p_y$ are the Cartesian pupil coordinates and $ω$ is the angular frequency of the light. The lens focusing coordinate systems in the pupil and focus are shown...
in Fig. 1(a). By solving the diffraction integral with a further small angle approximation [25], the field distribution \( E(\omega, x, y, z_f) \) in the focal plane is obtained with a 2D Fourier transform as

\[
E(\omega, x, y, z_f) = \frac{\omega}{i2\pi c} \exp(ik f) \exp \left[ \frac{ik \sqrt{x^2 + y^2}}{2f} \right] \times F[E(\omega, p_x, p_y)],
\]

where \( c \) is the speed of light, \( k \) the wavenumber \( (k = \omega/c) \), and \( f \) is the focal length of the lens.

The field distribution of each monochromatic wavelength element is calculated in and around the focal plane. After inverse Fourier transform from the spectral domain to the time domain, the intensity distribution can be calculated by

\[
I(t, x, y, z) = |E(t, x, y, z)|^2 = |F[E(\omega, x, y, z)]|^2.
\]

Spatiotemporal focusing generates various spatial intensity distributions at the focal plane by adopting different forms of pupil illumination. Two common modes of SSTF are line focusing, which is mostly adopted in scanning microscopy [6,7], as well as wide-field focusing, which is applied in laser fabrication [12,15,16] and provides fast optical sectioning images at a high frame rate for wide-field microscopy [28–30]. These modes employ different pupil illumination profiles and, as such, the sensitivity to the aberrations varies. As an axially confined line focus can be generated using either SSTF or conventional SF, we can first use this focusing mode to compare directly the relative susceptibility to aberrations of the two focusing principles. Then we move on to discuss the properties of wide-field SSTF in the presence of aberrations.

A difference between SSTF and SF is the distribution of each frequency component in the pupil of the objective lens. The frequency components are spatially spread along the pupil for SSTF, while combined together as a line for SF, as shown in Fig. 1. For line SSTF focusing, each spectral component is located at a different position along the \( p_x \) direction and fills the entire pupil along the \( p_y \) direction [Fig. 1(b1)]. We can represent such pupil illumination by [11]

\[
E(\omega, p_x, p_y) = E_0 \exp \left[ -\frac{\omega^2 (\omega - \omega_0)^2}{4} \right] \exp \left[ -\frac{(p_x - p_{\omega 0})^2}{s_{\text{BFP}}^2} \right] u(p_x, p_y),
\]

where \( E_0 \) is a constant, \( \tau \) is the pulse width of the input Gaussian distributed light, \( \omega_0 \) is the central frequency, \( s_{\text{BFP}} \) represents the physical dimension to which each spectral component is focused (the distance for the amplitude to drop to 1/e of the peak value) at the back focal plane of the objective lens, and \( p_{\omega 0} \), which is a function of \( \omega \) (proportional to \( (\omega - \omega_0) \)), represents the central wavelength position for each spectral component. \( u(p_x, p_y) \) reflects the finite extent of the objective lens pupil, where \( R \) is the pupil radius.

In wide-field SSTF, the pulse is spectrally spread along the \( p_x \) direction, while being spatially confined in the \( p_y \) direction [Fig. 1(c)] in the back focal plane of the objective, which can be represented by

\[
E(\omega, p_x, p_y) = E_0 \exp \left[ -\frac{\omega^2 (\omega - \omega_0)^2}{4} \right] \times \exp \left[ -\frac{(p_x - p_{\omega 0})^2 + p_y^2}{s_{\text{BFP}}^2} \right] u(p_x, p_y).
\]

In both Eqs. (3) and (4), \( p_{\omega 0} \) represents the spatial spreading of different frequencies in the pupil, and it is important in control of the axial and lateral extent of the focal intensity distribution. The value of \( p_{\omega 0} \) is effectively set by the optical system prior to the objective lens, such as a combination of a diffraction grating and a tube lens. In our calculation, we adjust \( p_{\omega 0} \) by assuming that the spectral component on the edge of the pupil has a field amplitude of 50% compared to the amplitude of the central frequency at the center of the pupil, which is \( E(\omega_{\text{edge}}, p_x = 1, p_y = 0) = 0.05 \times E(\omega_0, p_x = 0, p_y = 0) \). This setting was found theoretically to provide an optimum compromise between high axial resolution and high focal intensity.

The value of \( s_{\text{BFP}} \) affects the lateral distribution of the focus for both line and wide-field focus, with a smaller value leading to a greater lateral spread of the focus. The value of \( s_{\text{BFP}} \) is also controlled by the optical system prior to the objective lens. In our calculation, we set it to be 0.06 times the pupil radius. The values of \( p_{\omega 0} \) and \( s_{\text{BFP}} \) are chosen to provide realistic axial and lateral resolution, while permitting clear
representation of the distortion due to aberrations. The qualitative effects caused by the aberrations will be similar with different values of \( p_{0} \) and \( s_{BFP} \).

For conventional line SF [Fig. 1(b2)], the distribution in the back focal plane of the objective lens is represented by

\[
E(\omega, p_{x}, p_{y}) = E_{0} \exp \left[ -\frac{\omega^{2}(\omega - \omega_{0})^{2}}{4} \right] \exp \left[ -\frac{p_{x}^{2}}{s_{BFP}^{2}} \right] u(p_{x}, p_{y}).
\]

(6)

in which all the frequency components are combined together as line in the pupil. For a better comparison for the two line focusing methods, line SF’s pupil illumination beam size is set to be equal to line SSTF’s monochromatic beam size, which is \( s_{BFP} \).

In our simulations, the objective lens is assumed to be 60× 1.4 NA oil immersion. The central wavelength of the input light is 790 nm and different pulse widths will be considered. The dispersion of the objective lens is for simplicity not included in this analysis. The present modeling is based on scalar rather than vector theory. A comparison of SSTF using both theories was presented in [11]. Our modeling results show, however, that vectorial effects are not significant with the parameter settings used in this paper.

3. EFFECTS OF ABERRATIONS

A. Definition of Phase Aberrations

A phase function \( \phi \) in the pupil of the objective can be used to represent the aberrations arising from focusing into the specimen, such that the aberrated pupil function becomes

\[
E'(\omega, p_{x}, p_{y}) = E(\omega, p_{x}, p_{y})e^{-j\phi(\omega, p, \theta)}.
\]

(7)

The wavefront phase \( \phi \) can be represented by the expansion of an infinite sum of weighted orthogonal functions.

The Zernike circle polynomials are often used in optics as they form an orthogonal set of functions defined over a unit circle and have simple properties of invariance [31]. An aberrated wavefront can be expressed as the weighted sum of Zernike polynomials, defined in polar coordinates as

\[
\phi(\omega, \rho, \theta) = \sum_{k=1}^{\infty} c_{k}(\omega)Z_{k}(\rho, \theta),
\]

(8)

where \( c_{k} \) is the modal coefficient, \( \rho \) and \( \theta \) are the polar coordinates in a unit circle describing the same pupil as \( (p_{x}, p_{y}) \). As SSTF and pulsed-SF contain various different frequencies, the magnitude of the Zernike modes for each spectral component is defined as

\[
c_{k}(\omega) = c_{k}(\omega_{0}) \times \omega/\omega_{0}.
\]

(9)

This model accounts for aberrations that are induced by the propagation of the light through specimens with an inhomogeneous refractive index. Neglecting any dispersion within the specimen, the phase aberration is proportional to the frequency of the light.

Some low-order Zernike polynomials correspond to aberrations such as astigmatism, coma, and spherical aberration that are commonly encountered in most optics systems. In this paper, we analyze the aberration effects of astigmatism (Zernike mode 5, 6), coma (Zernike mode 7, 8), trefoil (Zernike mode 9, 10), and spherical (Zernike mode 11). A mathematical description of these modes is included in the Appendix A. This set of modes is sufficient to illustrate the effects of different aberration types. Note that the lower-order Zernike modes (piston, tilt, tilt, and defocus) are not included as they have either no effect on the focusing process (piston), or represent displacement of the focus (tip, tilt, defocus).

A very common aberration that we also consider arises from a planar mismatch in refractive index when light is focused into a uniform medium. Even a small refractive index difference between the objective lens immersion medium and the sample can lead to a significant spherical aberration [18]. The index mismatch between the immersion medium of the objective lens \( (n_{1}) \) and the specimen \( (n_{2}) \) is characterized by introducing a phase aberration function \( \phi(d, \rho) \) into the pupil function of the imaging lens. \( \phi(d, \rho) \) is defined as [18,20,32]

\[
\phi(d, \rho) = -k \cdot d \cdot NA \left( \sqrt{\frac{n_{2}^{2}}{NA^{2}} - \rho^{2}} - \sqrt{\frac{n_{1}^{2}}{NA^{2}} - \rho^{2}} \right).
\]

(10)

where \( k \) is the wavenumber, and \( d \) is the nominal focusing depth of the light in the specimen (the position of the geometrical focus, if the rays were not refracted at the interface). The aberration caused by the refractive index mismatch not only axially shifts the focus to a greater depth (if \( n_{2} > n_{1} \)), but also causes a distortion of the intensity distribution at the focus. In this paper, we are mainly interested in the focal distortion, so the phase that causes the axial shift is removed in the modeling [20]. An explanation of how the defocus element is removed from the index mismatch phase can be found in Appendix B.

B. Line Focusing

1. Temporal Intensity Distribution

When considering time averages, SSTF and SF generate the same line shape in the focal plane, albeit through different processes and principles. Line SSTF is created by using a spatially chirped beam at the back focal plane of the objective lens [13,14]. In effect, this creates a single focal spot at one instant time, and the position of the focal spot shifts along the \( x \) axis temporally. This apparent motion of the focus in the \( x \) direction is a consequence of the PPT. The first column in Fig. 2(a) shows the temporal distribution of the focal point for SSTF, at the time of −1.5, 0, and 1.5 ps, relative to the time of maximum focal intensity using 150 fs pulsed laser light illumination. It is clear that the focal point shifts along the \( x \) axis, and the intensity increases to the peak at the central focus, and then decreases when it moves away. The time average \( x-z \) profile shows an obvious line along the \( x \) axis. Conventional SF uses spectrally uniform illumination, which directly generates a line along the \( x \) axis in the focal plane. This focal shape is a consequence of the reduced NA in one dimension (along \( p_{x} \)) of the objective pupil. The line exists at all points in time (−0.15, 0, and 0.15 ps) with different intensity, as shown in the first column of Fig. 2(b).

Aberrations affect the line focusing both spatially and temporally. As an example, we consider the aberration arising from a mismatch in the refractive index between an objective immersion \( (n_{1} = 1.52) \) and a diamond sample \( (n_{2} = 2.4) \),
which is relevant for laser machining applications [23]. The second columns in Figs. 2(a) and 2(b) show the cases when the laser light is focused into diamond with a nominal focusing depth of 25 μm. It can be seen that the temporal shapes of both SSTF and SF are distorted and the peak intensity points shift. Sidelobes are generated at lower depths beside the peak intensity points. We note that the sidelobes of SF are larger than that of SSTF, which can be seen in both temporal and time average profiles.

These results were calculated for specific conditions, but the qualitative effects are similar for other focusing configurations and materials. In general, there are some scenarios where the temporal profile at the focus needs to be considered; however, the time average result will be more important for most applications.

2. Zernike Mode Aberration

In this section, we study the effects of lower-order Zernike modes on the properties of line focusing systems. All the profiles are calculated as the time average of the squared intensity \(I(t^2) = 1/T \int_0^T I(t)dt\), where \(T\) is the repeat period of the laser light, which corresponds to the excitation probability for a second-order process, such as two-photon microscopy. Although in general other nonlinear processes may be used, this quantity provides adequate illustration of the focusing effects. For the line focus, the \(I(t^2)\) distribution along the \(y\) and \(z\) direction is studied (at the plane where \(x = 0\)), because it gives a better view of the distortion both along axial and lateral directions. The \(y-z\) \(I(t^2)\) profile is perpendicular to the focusing line, and has similar profile for small values of \(x\).

Figure 2 shows the distortions occurring in SSTF (a)–(c) and SF (d)–(f) in the presence of individual Zernike modes. The SSTF pupil illumination profile is shown in the first inset in (a); plots of the Zernike modes in the pupil are shown as the remaining insets. While the unaberrated line focus stretches as a line along the \(x\) direction, it has a good confinement along both the \(z\) and \(y\) direction, which are shown as the first profile in (a). The \(I(t^2)\) profiles show different distortions when Zernike modes are considered. Mode 6 (astigmatism) and mode 11 (spherical) cause the peak \(I(t^2)\) to shift along the axial direction, while other modes, especially mode 8 (conic), mode 10 (trefoil), and mode 11 (spherical), produce various amounts of focal distortion. These effects are also characterized by the plots in row (b), which show how the peak \(I(t^2)\) in the \(y\) direction changes along the \(z\) direction (where we define \(I_p(z) = \text{Max}_y[I(t^2)(y, z)]\)). The effects of particular Zernike mode aberrations can be understood by comparison of the pupil illumination profile and phase plots [the insets in row (a)] in each case. For example, mode 6 creates a defocus-like phase distortion for each spectral component in the pupil, which shifts the focus for each component by a different amount along the \(z\) axis; modes 8, 10, and 11 cause more complex phase deviations for each spectral component, which leads to distortion at the focus. The peak \(I(t^2)\) of the distorted focus as a function of the amplitude of the Zernike mode [measured in radians (rms) at the central wavelength] is also illustrated in row (c). As the SSTF uses full pupil illumination, all the Zernike modes have effects on the focusing, thus causing reduction in the peak \(I(t^2)\).

A comparison of these results with line SF is shown in Figs. 3(d)–3(f). Similarly, mode 8 (conic), mode 10 (trefoil), and mode 11 (spherical) produce various amounts of intensity distortion. As shown in row (f), these three modes cause a drop in the peak intensity. From the \(I(t^2)\) profiles and the axial \(I_p(z)\) distribution, it is seen that the sidelobes of the distorted SF are stronger than that of SSTF. The larger drop in peak \(I(t^2)\) is because the pupil illumination of SF is a single line, and the Zernike modes 8, 10, and 11 have a more significant phase distortion along this line than in other areas. Mode 6 (astigmatism) and mode 11 (spherical) cause a focus shift along the \(z\) direction, but mode 6 exhibits a very slight drop in peak intensity. By comparing the pupil illumination of line SF and the phase plots of each Zernike mode, it is easy to find that modes 5, 7, and 9 have little effect on the line SF, because the phase in these three modes is nearly flat in the area along line illumination in the SF pupil. Through these investigations, we can conclude that, the value of \(I(t^2)\) in SSTF line focusing is more sensitive to certain Zernike aberrations than the equivalent value for SF line focusing. However, the effects on spatial resolution are similar in both cases.

3. Index Mismatch Aberration

In many laser fabrication and microscopy applications, a high NA objective lens is used to focus laser light deep into a sample with a refractive index differing to the lens immersion medium. This scenario is modeled in Figs. 4(a) and 4(b) for both SF and SSTF, where we consider the aberration arising when the light is focused with a 1.4 NA oil (\(n_1 = 1.52\)) lens into a diamond sample (\(n_2 = 2.4\)) with a nominal focusing depth of 50 μm, focusing from the top of the profile [23]. The focus is distorted generating several sidelobes above the peak intensity point, with a corresponding reduction in peak \(I(t^2)\). Figure 4(c) shows the \(I_p(z)\) distribution along the axial direction for unaberrated and aberrated line SSTF as well as line SF foci, and each curve is individually normalized. The FWHM of the \(I(t^2)\) along the \(z\) direction becomes wider, and the on-axis peak intensity points are obviously shifted because of the distortion. We note that this shift
The on-axis peak intensity is in addition to the axial shift due to the defocus component of the phase aberration function, described in Subsection 3.A. An explanation for this additional focal shift is given in Appendix B. These aberration phenomena have been widely observed in modeling of refractive index mismatch induced aberrations in conventional focusing systems [32]. The \( F^2 \) profiles and curves show that the side-lobes of SSTF line focusing are smaller than that of SF. Figure 4(d) shows the peak \( F^2 \) dropping with the increase in nominal focus depth into the sample. As a conclusion, the effect of index mismatch is slightly smaller on line SSTF than that on line SF, which suggests that line SSTF might be a better choice for the cases involving deep focusing into high index sample considering the amount of aberration.

C. Wide-Field Focusing

1. Zernike Mode Aberrations

In the wide-field SSTF, a symmetric focusing shape is obtained both along the axial and lateral direction, which is shown by the \( x-y \) profile in Fig. 1(c) and unaberrated \( x-y \) profile in Fig. 5(a). The wide-field area experiences different peak intensity shifts or intensity stretches when Zernike mode
aberrations are considered. The $x$–$z$ ($I^2$) profiles (at $y = 0$) in the presence of Zernike modes 5–11 are shown in Fig. 5(a), the variation of $I_p(z)$ is presented in Fig. 5(b), and Fig. 5(c) shows how peak ($I^2$) changes with each Zernike mode's amplitude. The pupil illumination of wide-field SSTF is such that the components of the spectrum are distributed along the $p_x$ direction, but only have a narrow extent in the $p_y$ direction. Comparing the pupil illumination to each Zernike mode plot in the pupil (the insets in Fig. 5(a)), it is understandable that modes 5, 8, and 10 have little effect on the focusing properties. The input energy is spread as a line along the $p_x$ axis in the pupil, where, for these modes, the phase is essentially constant. The axial ($I^2$) distribution is nearly the same as the unaberrated profile, and the peak ($I^2$) shows negligible change with increasing mode amplitude. Mode 6 slightly shifts the axial position of the wide-field area, but does not cause intensity reduction; this is a consequence of the quadratic, defocus-like form of phase along the illumination axis of the pupil. Modes 7, 9, and 11 distort the focus, and cause a decrease in focal intensity. It is notable that mode 11 (spherical) not only stretches the focus, but also causes large shift in peak intensity along the axial direction.

2. Index Mismatch Aberration

When the laser light is focused in wide-field SSTF by a high NA lens deep into a high index sample, the index mismatch causes focal distortion and consequently a reduction in axial resolution. Comparison of the unaberrated $x$–$z$ ($I^2$) profile in Fig. 6(a) and the index mismatch aberrated profile in Fig. 6(b) indicates an obvious change in the shape and the position of the focusing area. The ($I^2$) distribution along the axial direction for the two profiles is shown in Fig. 6(c). We find the FWHM increases for aberrated focusing. The FWHM increases from 1.76 μm at the surface to 3.26 μm at a nominal focus depth of 50 μm. The axial FWHM of the ($I^2$) distribution as a function of nominal focus depth into the sample is shown in Fig. 6(d). The FWHM continues to broaden with increasing depth, with a corresponding reduction in peak intensity, as shown in Fig. 6(e). It is notable that with the same focusing depth into a sample, the peak intensity of wide-field SSTF is not as sensitive to aberrations as the line focus [Fig. 4(d)]. For example, when the light is focused into diamond with a depth of 50 μm, the peak ($I^2$) dropped to 61% for wide-field SSTF, 7% for line SSTF, and 4% for line SF.

D. Effects of Aberrations and Pulse Width on Focusing

As illustrated in Eq. (9), the Zernike mode amplitude is a parameter dependent on frequency, while the spectral range of the laser light is set by the pulse duration, suggesting that the pulse duration should have an influence on the effects of aberrations. The qualitative effects are similar for different pulse durations, but the level of distortion varies for each specific focusing method. We take the example of coma aberration to illustrate the effects of changing the pulse duration.

Fig. 5. Effects of Zernike mode aberrations on wide-field SSTF. (a) Comparison of ($I^2$) $x$–$z$ profiles between unaberrated and Zernike mode 5–11 aberrated [central wavelength amplitude equals to 1 rad (rms)] wide-field SSTF. The inset in the first figure is the pupil illumination, and the insets in the aberrated figures are the Zernike modes phase plot in the pupil. (b) Normalized plot of the variation of $I_p(z)$ (red curve). The black curve is the unaberrated version included for comparison. (c) Peak ($I^2$) changes with Zernike mode amplitude [radians (rms)] of the central wavelength.

Fig. 6. Effects of refractive index mismatch on wide-field SSTF. The sample is diamond with the refractive index of 2.4. The nominal focus depth is 50 μm. (a) Unaberrated ($I^2$) $x$–$z$ profile for wide-field SSTF. (b) Aberrated ($I^2$) $x$–$z$ profile. Insets are the pupil illumination (left) as well as the phase induced by the index mismatch (right, with defocus element removed). (c) Comparison of axial ($I^2$) distribution for unaberrated and index mismatch aberrated wide-field SSTF. Each curve is individually normalized. (d) FWHM of the ($I^2$) distribution along axial direction versus nominal focus depth into the sample. (e) Peak ($I^2$) changes with the focusing depth into the sample for SSTF.
Peak \( \langle I^2 \rangle \) versus Zernike mode amplitude is calculated for different pulse widths, while \( p_{x,0} \) is adjusted for each width according to the description in Section 2 in order to maintain the same axial and lateral resolution. Figure 7 shows the influence of the pulse width on line SSTF and wide-field SSTF. Focusing with shorter pulse durations is slightly more affected by the aberrations. For example, in Fig. 7(b), when the coma aberration is applied with amplitude of 1 rad (rms) at the central wavelength, the peak \( \langle I^2 \rangle \) using 300 fs pulses drops to 80%, but for 5 fs pulse the peak \( \langle I^2 \rangle \) drops to 72%.

### 4. CONCLUSIONS

Phase aberrations affect the focal distributions in SSTF and SF systems both temporally and spatially. This has been shown for lower-order Zernike modes and refractive index mismatch aberrations in line and wide-field focusing systems. The particular aberrations cause reduction of the axial and lateral resolution, reduction in peak intensity, or shift of the focus. Compared to line SF, line SSTF is more sensitive to Zernike modes aberrations, while slightly less sensitive to index mismatch aberrations. Wide-field SSTF is much more robust to both Zernike modes and index mismatch aberrations than line focusing.

The reason for the robustness of line SF and wide-field SSTF to some Zernike modes, for example modes 5, 7, and 9 for line SF and modes 5, 8, and 10 for wide-field SSTF, is that only part of the pupil is illuminated. The focus will be less distorted if this illuminated part of the pupil has less phase aberration for particular Zernike modes. We note that one can use this observation to draw conclusions about other modes that we have not studied directly here.

When the laser light is focused into a sample with a refractive index mismatch, the FWHM is broadened and peak intensity is reduced. These trends for either line or wide-field SSTF are also applicable for other different materials and lenses, other than used for the calculations in this paper.

We should also note that in this paper, we mainly use a second-order excitation probability to model focal aberration effects. However, the trends observed will also be relevant for higher-order processes, such as in harmonic generation for microscopy or high-order fabrication processes.

These results provide the basis for understanding the effects of aberrations on SSTF systems and the performance improvement that might be achieved through aberration correction. Several SSTF systems now include complex illumination structures to create photoactivation patterns [9,10], or for patterned laser fabrication [15,16]. Further work will illustrate how aberrations affect these more complex illumination systems. Furthermore, we also note that in applications of SSTF multiphoton microscopy, imaging of the generated fluorescence onto the detector will also be affected by the aberrations, whereas our modeling so far has involved only the excitation effects.

### APPENDIX A: ZERNIKE POLYNOMIALS

The Zernike modes from 5 to 11 are shown in Table 1. The azimuthal frequency \( m \) and radial polynomials of degree \( n \) [31] are also shown. Orthonormal Zernike polynomials have been normalized so that each polynomial has a RMS value of 1 rad over the unit circle, which is defined by the hard edge aperture found in objective lenses.

### APPENDIX B: INDEX MISMATCH PHASE CALCULATION

Equation (10), describing the index mismatch between the immersion medium and the sample, contains both a defocus element (causing an axial shift of the focus) and aspherical aberration element (causing focal shape distortion). This appendix presents the calculation method to obtain “defocus-free” phase by removing the defocus element [20]. Practical aberration correction usually removes any focal distortion without refocusing the system. This is useful to maintain the dynamic range for any AOE. With a high NA objective lens, we employ a spherical term rather than a quadratic term to describe the defocus phase element:

\[
D(d, \rho) = k \cdot d \cdot NA \sqrt{\frac{n_2^2}{NA^2} - \rho^2}.
\]

The process to obtain defocus-free phase \( \hat{\phi}(d, \rho) \) can be expressed as

\[
\hat{\phi}(d, \rho) = \phi(d, \rho) - \frac{\phi'(d, \rho) D'(d, \rho)}{D(d, \rho) D'(d, \rho)} D(d, \rho).
\]

\( \phi'(d, \rho) \) and \( D'(d, \rho) \) represent spherical aberration and defocus with subtracted mean values,

\[
\phi'(d, \rho) = \phi(d, \rho) - \frac{1}{N} \sum_{\rho} \phi(d, \rho)
\]

\[
D'(d, \rho) = D(d, \rho) - \frac{1}{N} \sum_{\rho} D(d, \rho).
\]

where \( N \) is the total number of the calculation pixels. The calculation \( \left( \ldots \right) \) defines an inner product over the pupil:

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**Table 1. Zernike Polynomials \( k = 5 \) to \( 11 \)**

<table>
<thead>
<tr>
<th>( k )</th>
<th>( n )</th>
<th>( m )</th>
<th>( Z_{k}(\rho, \theta) )</th>
<th>Aberration Term</th>
</tr>
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<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>( 2\sqrt{3} \rho^2 \cos(2\theta) )</td>
<td>Astigmatism</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>−2</td>
<td>( 2\sqrt{3} \rho^2 \sin(2\theta) )</td>
<td>Astigmatism</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
<td>( 2(3\rho^4 - 2\rho^2) \cos(\theta) )</td>
<td>Coma</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>−1</td>
<td>( 2(3\rho^4 - 2\rho^2) \sin(\theta) )</td>
<td>Coma</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>( 2\sqrt{3} \rho^4 \cos(3\theta) )</td>
<td>Trefoil</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>−3</td>
<td>( 2\sqrt{3} \rho^4 \sin(3\theta) )</td>
<td>Trefoil</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>0</td>
<td>( \sqrt{3}(\rho^6 - 3\rho^4 + 1) )</td>
<td>Spherical</td>
</tr>
</tbody>
</table>
\[
    \langle \phi'(d, \rho), D(d, \rho) \rangle = \frac{1}{N} \sum_{\rho} \phi'(d, \rho) \cdot D(d, \rho).
\]  

(B4)

Figures 2, 4, and 6 show additional shifts in the on-axis peak intensity even after removal of this defocus component. The reason for this additional shift is that the defocus removal relies upon the minimization of the rms phase error, which does not necessarily mean the maximum intensity point resides in the nominal focal plane for large aberrations.

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REFERENCES

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