Discontinuous Galerkin methods on polygonal and polyhedral meshes

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Abstract:

Numerical methods defined on computational meshes comprising of polygonal and/or polyhedral (henceforth collectively termed “polytopic”) elements, with, potentially, many faces, have gained substantial traction recently for a number of important reasons. A key underlying issue is the design of a suitable computational mesh upon which the underlying PDE problem will be discretized. The task of generating the mesh must address two competing issues. The mesh should provide a good representation of the given computational geometry with sufficient resolution for the accurate approximations. On the other hand, the mesh should not be too fine, so that computational complexity becomes prohibitive due to the high number of numerical degrees of freedom. Standard mesh generators output grids consisting of triangular/quadrilateral elements in 2D and tetrahedral/hexahedral/prismatic/pyramidal elements in 3D. In the presence of essentially lower-dimensional solution features, for example, boundary/internal layers, anisotropic meshing may be exploited. However, in regions of high curvature, the use of such highly-stretched elements may lead to element self-intersection, unless the curvature of the geometry is carefully ‘propagated’ into the interior of the mesh through the use of (computationally expensive) isoparametric element mappings. These issues are particularly pertinent in the context of high-order methods, since in this setting, accuracy is often achieved by exploiting coarse meshes in combination with local high-order polynomial basis functions. I will argue that, by dramatically increasing the flexibility in terms of the set of admissible element shapes present in the computational mesh, the resulting discontinuous Galerkin FEMs can potentially deliver dramatic savings in computational costs. Moreover, if time permits, I will present some recent theoretical developments in the error analysis of such methods.